

## Routing of Railway Systems with the S-graph Framework for Effective Scheduling

Róbert Adonyi<sup>1\*</sup>, István Heckl<sup>1</sup>, Andrew Szalamin<sup>2</sup>, Ferenc Olti<sup>1</sup>

<sup>1</sup>University of Pannonia, Department of Computer Science and Systems Technology,  
Egyetem u. 10., 8200 Veszprém, Hungary  
adonyi@dcs.uni-pannon.hu

<sup>2</sup>Bamco Kkt., Somogyi út 44., 1110 Budapest;

In the present work, the S-graph framework has been adopted for solving the routing and scheduling of railway systems. The proposed method can be effectively integrated into the overall supply chain management, including production and distribution systems.

The scheduling of a large-scale production system with batch and continuous operations is a complex optimization problem. The formerly developed S-graph framework (Sanmarti et. al. 1998, 2002) is proven to be highly effective for multi-purpose batch scheduling. This framework has been extended to handle the routing of railway systems. Optimal scheduling of a railway system becomes extremely complex if a large number of trains are considered. In most cases, however, the search space of the optimization procedure can be reduced substantially by eliminating redundant solutions through additional combinatorial constraints. The proposed methodology aims to achieve this reduction.

The problem is represented by a directed graph, which can handle more than one equipment unit with different working and changeover times to perform the same operation. An algorithm has been developed to exploit this specific property. A tailored branching strategy has been invented to determine an optimal or near optimal solution using an effective algorithm for sharp bounding.

The method has been implemented in C++ and tested extensively. The efficacy of the new approach is demonstrated by the scheduling of an underground railway system. These trains are delivering perishable commodities (e.g. concrete), thus, the optimal schedule greatly contributes to waste reduction.

### 1. Introduction

The vehicle routing problem (VRP) is a combinatorial optimization problem which is widely studied in literature (Toth and Vigo, 2002). Routing trains through railway networks and stations is a special case of the vehicle routing problem. The detailed layout of the network within the stations and between neighboring stations is given.

Underground construction is a resource demanding and expensive industrial undertaking, where planning and optimization play an important role. The optimization

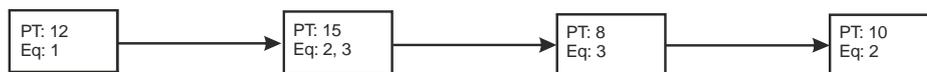
of the construction process is interconnected with different combinatorial problems, e.g., supply chain optimization and scheduling. During underground construction, huge amounts of construction materials are moved and processed. The construction consists of the tunnel and the station buildings. In this paper, the tunnel construction part is detailed, especially in consideration of the logistic processes.

The running tunnels are constructed by shield tunneling technique using so-called tunnel boring machines (TBM). The tunnel wall, which consists of concrete segments, can be assembled ring by ring within the shield plate. The rings are connected with bolts and a rubber or plastic sealing system that can immediately provide a waterproof wall. The annular gap around the tunnel is filled with cement mortar.

The TBMs consume huge amounts of construction materials. The required material is transported with a given number of supply trains from the depot to the TBMs. The traffic rules (e.g. the speed limits) are given for the trains and the network. The trains are transporting perishable materials (e.g. mortar, concrete), therefore the minimal time routing is very important to avoid waste. Optimal scheduling also ensures continuous supply at the TBMs, increasing their efficiency and thereby decreasing overall energy consumption. In this paper, the S-graph framework is extended for the scheduling and routing of the supplying trains of the TBMs considering the minimal duration time routing for the trains.

## 2. S-graph framework

The S-graph framework developed by Sanmartí et al. (1998, 2002) aims at the short term scheduling of multi-purpose batch plants. The framework includes a representation, a mathematical model, a solution method called the basic algorithm, and acceleration tools. The basic idea is to consider a problem formulation that manifests the unique structure of the class of scheduling problems and a solution procedure that exploits the specific structure of the problem (Friedler 2009, 2010). This approach can result in the following advantages: (1) globally optimal solutions are obtained, (2) no infeasible solutions are found in terms of cross-transfers, (3) search space is significantly reduced, and (4) it consists of a continuous formulation without the necessity of determining the so-called time points.



*Figure 1. Conventional representation of a recipe.*

The definition of a scheduling problem is the following: a number of products are to be produced in the shortest possible time (minimizing the makespan). The production of each product is given by a recipe (see Fig. 1) which contains the ordering of the tasks belonging to a given product and the set of equipment units capable to perform the specific tasks. Usually there are more tasks than equipment units; therefore, the equipment units have to be scheduled. The schedule of an equipment unit determines the tasks to be performed by that unit, the order of these tasks, and possibly timing information as well.

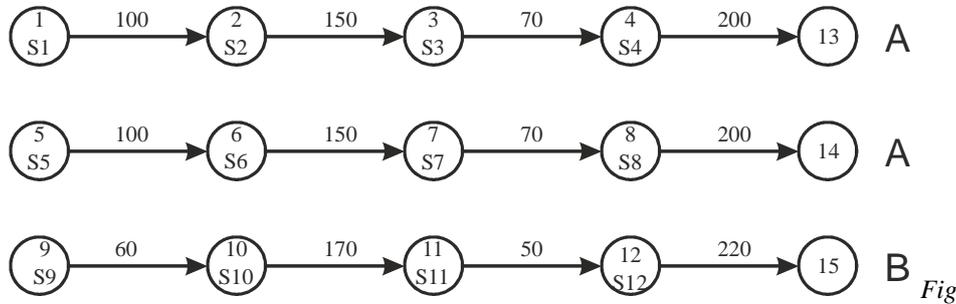


Figure 2. Recipe-graph of the illustrative example: two batches of product A and one batch of product B.

Each task given in the recipe is represented by a node in the S-graph. Moreover, an additional node is assigned to each product (see Fig. 2). The set of those equipment units that can perform task  $i$  is denoted by  $S_i$ . The processing order of the tasks is given by the arcs of the graph (recipe-arcs). The processing time of potential equipment units for each task is known. The minimum processing time is assigned for each recipe arc. In this representation, the value assigned to an arc expresses a lower bound for the difference of the starting times of the two related tasks. Formally, an S-graph is given in the form of  $G(N, A_1, A_2)$ , where  $N, A_1, A_2$  denote the set of nodes, the set of the recipe-arcs and the set of schedule-arcs, respectively.

For generating multiple batches of the products, the appropriate part of the S-graph is repeated according to the number of batches to get the recipe-graph of multiple batches, see Fig. 2. The acceleration tools of the S-graph framework ensure that search space does not increase unnecessarily in this case.

The S-graph can represent different storage policies including non-intermediate storage (NIS), unlimited intermediate storage (UIS), zero waiting (ZW), and common intermediate storage (CIS) policies. The NIS policy is particularly difficult to handle with mathematical programming models as the cross-transfer which is prohibited with NIS cannot be eliminated. In the following, this storage policy is our focus.

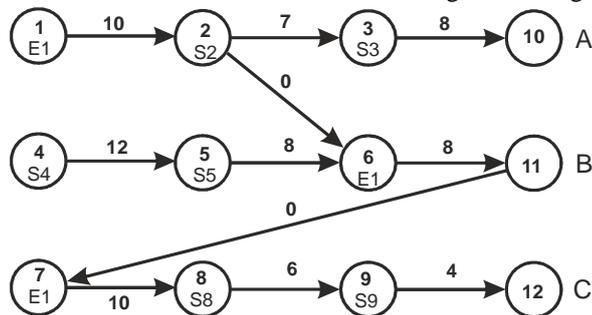


Figure 3. S-graph representation of task sequence 1-6-7 for equipment unit E1 with NIS policy.

The representation of the schedule-arcs depends on storage policy. With NIS policy, the intermediate materials of a schedule are always stored in the corresponding equipment unit. If equipment unit E1 is assigned to task 2 and consecutively to task 6, then an arc is established from all the consecutive tasks of task 2 to task 6 as shown in Fig. 3. The weight of the arc is equal to the length of the changeover.

A branch-and-bound (B&B) procedure is used for generating the optimal schedule of a scheduling problem. The recipe-graph, with no equipment unit assignment, serves as the root of the enumeration tree of the B&B procedure. With any partial problem, one equipment unit is selected and then all child partial problems are generated through the possible assignments of this equipment unit to unscheduled nodes. The scheduling of the selected unit is extended in the child partial problems and all possibilities are taken into account.

The bounding procedure tests the feasibility of a partial problem by searching for directed loops. If this test is positive, it determines the lower bound for the makespan of all solutions that can be derived from this partial problem by using the longest path algorithm. Any additional schedule arc can only increase the longest path.

### 3. Solving the routing problem with the S-graph framework

To use the combinatorial algorithms of the S-graph framework for the routing of supply trains, the VRP optimization problem has to be transformed to a batch process scheduling problem. The network of rails can be decomposed to regions by the underground stations. The regions are labeled R1 through R5 (see Fig. 4.). The supply trains start from the depot, pass through regions R1, R2, ..., R5, supply the TBM and return to the depot through regions R5 through R1. In this example, the stations are represented by regions R2 and R4. Within the regions, the sections of rails are identified 1 through 15. Considering the network of Fig. 4., the route of a train can be e.g., 1-3-7-12-15-10-8-6-1.

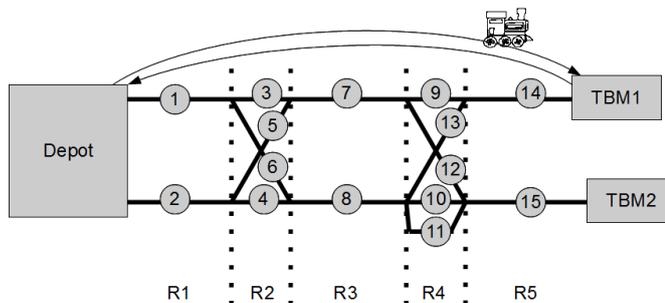


Figure 4. Railway network of the supply train routing problem.

The input of the scheduling problem is described by the recipe. A recipe contains a network of tasks, the set of equipment units for the tasks, together with processing and changeover times. The recipe in the routing problem should represent the overall movement of a supply train through the network. A task in this recipe corresponds to the location of a supply train inside a region. The equipment units correspond to the sections of rails.

If the routing problem is transformed to a scheduling one and is solved by the S-graph framework, the solutions can represent infeasible routings. This infeasibility derives from the potential connections of the sections of rails. For example, a supply train on rail 7 can continue its route only on rails 9 or 12, but not on rails 10, 11, or 13. This constraint comes from the infrastructure and has to be considered in the branching procedure, therefore a novel task-equipment unit assignment strategy is developed. If the NIS strategy is realized for the solution, the schedule can represent a feasible routing of trains. Because of the NIS strategy, only one train can reside simultaneously on any section of rails.

#### 4. Illustrative example

In this illustrative example, the minimal makespan schedule (routing) is determined for the network of Fig. 4 with three supply trains. Table 1 contains the minimal required times for the trains to pass along a section of rails.

Table 1. Time requirement to pass along a section.

Section	Time (min)	Section	Time (min)	Section	Time (min)
1	5.3	6	1.4	11	1.3
2	6.0	7	8.6	12	1.9
3	1.4	8	8.2	13	1.8
4	1.3	9	1.2	14	3.1
5	1.4	10	1.2	15	2.9

The optimal routing solution requires 44.7 min. The routes of the trains are indicated by the arrows on Fig. 5.

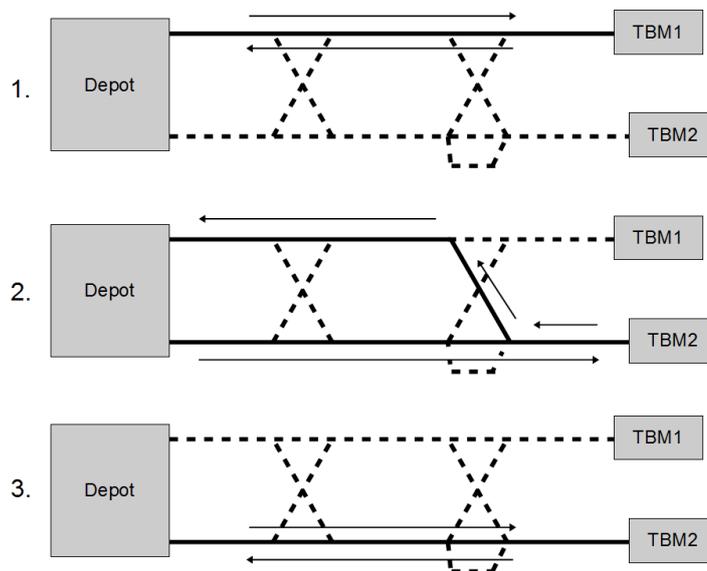


Figure 5. Optimal routes of the trains.

## 5. Conclusion

In this work, the formerly developed S-graph has been extended in order to handle the routing of railway systems. The solution of the S-graph problem gives a minimal time routing for the supply trains. The method has been implemented in C++ and tested extensively.

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