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Combinatorial Algorithm for Synthesizing Redundant Structures to Increase Reliability of Supply Chains: Application to Biodiesel Supply

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ABSTRACT: The current work reveals a methodology that provides an adequate basis to portray and model supply chains mathematically and formally as well as to synthesize optimal and alternative supply scenarios algorithmically while taking into account structural redundancy. The proposed methodology is based on the combinatorial foundations of algorithmic process synthesis or more specifically on the P-graph framework. A biodiesel supply network involving blending and transportation serves as an illustrative example. A novel algorithm generates the mathematical model and alternative solutions to increase reliability of supply scenarios. Major steps of the generation are the structure generation and estimation of reliability of a supply scenario.

■ INTRODUCTION

Rapidly changing prices and taxes, as well as emerging new products and technologies, demand that for any corporation to remain competitive the design of its supply chain has to be frequently revised. Mathematical modeling and optimization provide a solid basis for evaluating alternative designs and scenarios, reducing cost, improving efficiency, and adapting to changing requirements.

Several robust and reliable process optimization algorithms have been developed and implemented on the basis of the P-graph framework by Friedler et al.[1−3] The approach based on the P-graph framework appears to be the only one capable of executing process-network optimization giving rise to an algorithmically and mathematically proven solution for all steps involved, comprising superstructure generation and construction of the mathematical model,[1−4] as well as optimization, and the solution interpretation.[5]

The superstructure provided by P-graph algorithm is a rigorous superstructure as introduced by Kovács et al. in ref 6. Let a set of operating units and the mathematical model of each operating unit be given. Moreover, a systematic procedure is presumed to be available so that a valid mathematical programming model can be generated for a network of the given operating units. Then this network is deemed to be a rigorous superstructure for a class of process-synthesis problems if the optimality of the resultant solution cannot be improved for any instance of the class of problems by any other network of operating units and model generation procedure. According to the P-graph methodology the union of the combinatorially feasible structures called the maximal structure serves as the superstructure. Consequently, it contains each feasible and optimal structure as a part, thus proven to be a rigorous superstructure.

Although process-network synthesis involves hard combinatorial optimization problems,[7−9] P-graph-based algorithms and the related software are effective for enumerating feasible structural alternatives by algorithm SSG as well as determining the optimal or n-best solutions in light of various criteria by algorithm ABB, e.g., cost and reliability. Former examinations show that the P-graph approach to process-network synthesis (PNS) originally conceived for conceptual design of chemical processes,[10,11] optimal workflow structure,[12] and supply chain optimization[13] provides appropriate tools for generating and analyzing structural alternatives for product supply problems.[14] To adapt P-graph algorithms and software to the design and optimization of reliable supply chains, extension of the algorithms to properly consider structural redundancy is required.

Table 1. Inputs and Outputs of Activities

activities	inputs	outputs	reliability
blend (B)	K2 component, K4 component, K7 component, K8 component	biodiesel produced in Pécs	r_B = 98.00%
upload 1 (U1)	biodiesel in Dombóvár Depot	biodiesel in truck in Dombóvár	r_{U1} = 98.00%
upload 2 (U2)	biodiesel in Pécs Depot	biodiesel in truck in Pécs	r_{U2} = 98.00%
upload 3 (U3)	biodiesel produced in Pécs	biodiesel in truck in Pécs	r_{U3} = 97.00%
transport 1 (T1)	biodiesel in truck in Dombóvár	biodiesel in truck in Kaposvár	r_{T1} = 98.00%
transport 2 (T2)	biodiesel in truck in Pécs	biodiesel in truck in Kaposvár	r_{T2} = 98.00%
download (D)	biodiesel in truck in Kaposvár	biodiesel in Kaposvár	r_D = 99.00%

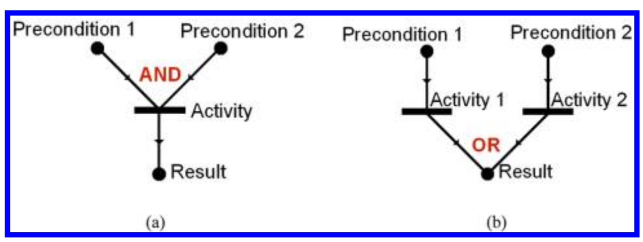

Figure 1. Logical AND (a) and OR (b) relations in P-graphs.

Redundancy, i.e., establishing and allocating capacities to perform activities in parallel instead of setting up key resources and bottle necks, is a natural way of improving reliability. Cost optimization, however, often results in eliminating redundancies. That is why generating design alternatives redundancy has to be

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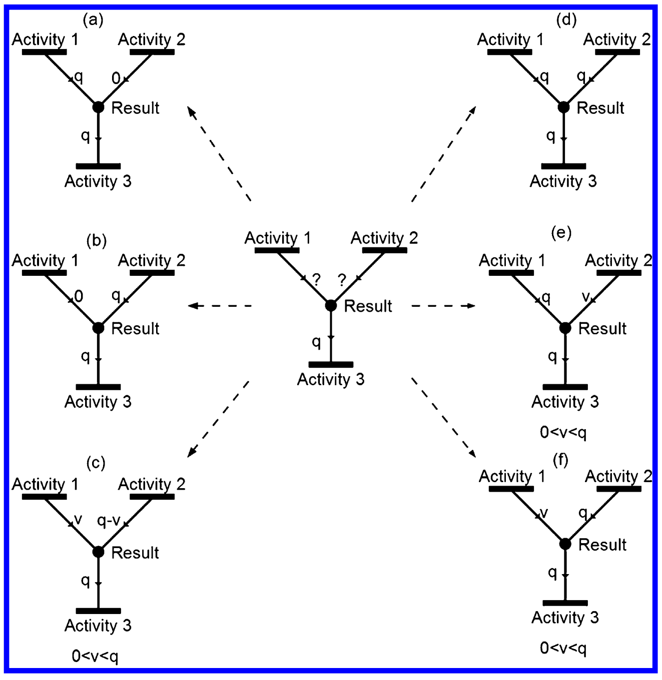

Figure 2. Structural representation of the alternative cooperation scenarios of two activities.

considered as an optional requirement before cost optimization. The current work introduces a combinatorial algorithm for extending the P-graph representing the superstructure for supply chain synthesis problem in order to potentially increase reliability. The extended superstructure incorporates steps that can guarantee redundancies regardless of the cost optimization. As a result, alternative scenarios generated and optimized on the basis of the extended superstructure involve every combination of the potentially redundant activities.

■ CASE STUDY

Because a final target can often be reached in different ways, the examination of reliabilities of the alternative scenarios, i.e., supply chain structures, has high practical importance. The more combinations of the chain elements in the supply structure are sufficient to reach the final targets, the higher is the overall reliability of the supply chain. A supply network synthesis problem illustrates the proposed methodology. An oil company operates plants at three locations which are the towns of Pécs, Dombóvár, and Kaposvár. The task to be performed is to satisfy biodiesel demands of the company's plant in Kaposvár from the other two locations with a minimal overall risk. A limited amount of biodiesel and its components are available in Dombóvár and Pécs. In Pécs, a limited capacity for blending biodiesel from available

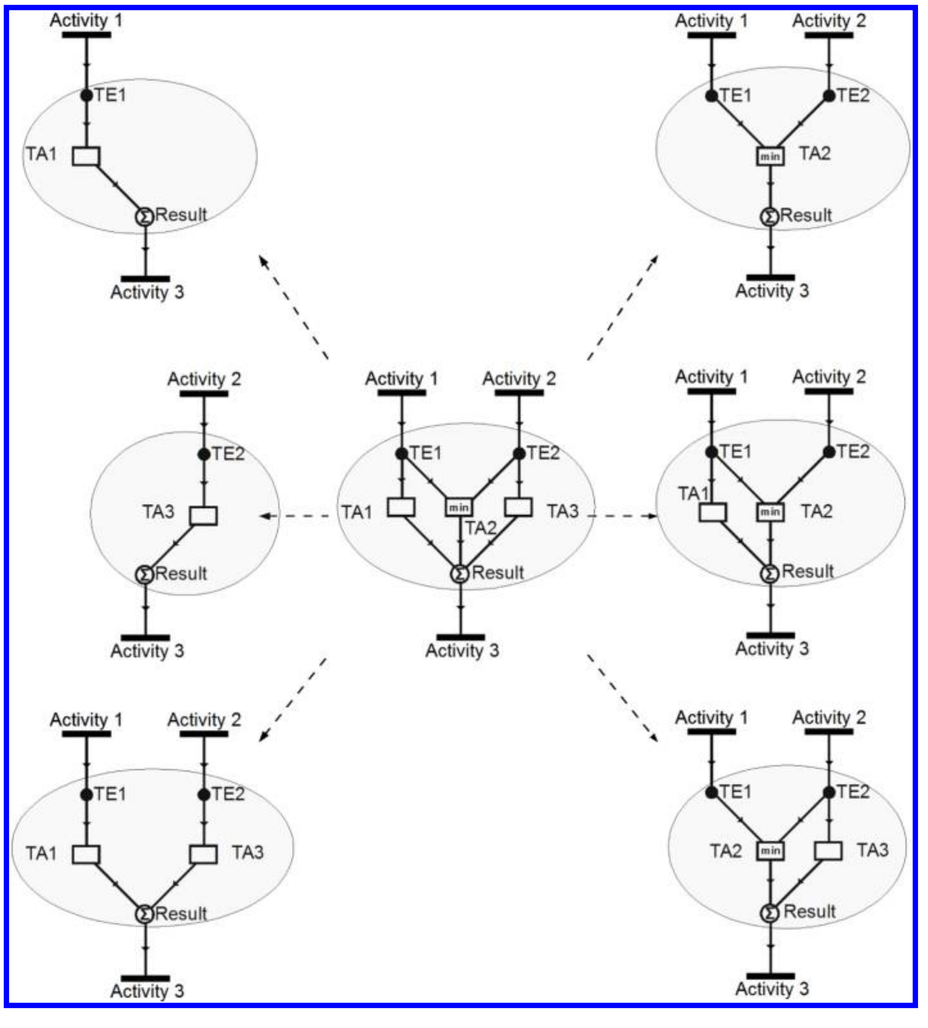

Figure 3. Superstructure of the alternative scenarios for two potentially parallel activities.

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Input: M, P, R, O sets (maximal structure);
Output: Redundancy extended maximal structure
begin
  $count := 0$;
  $m' := \varnothing$;
  $split := \bigcup_{m \in (\Psi^{+}(o) \cap \Psi^{-}(o)) \cup P \text{ and } |\varphi^{-}(m)| > 1} \{m\}$;
  while $split \neq \varnothing$ **begin**
    $x \in split$;
    $O_{out} = \varphi^{-}(\{x\})$;
    while $O_{out} \neq \varnothing$ **begin**
      $(\alpha, \beta) \in O_{out}$;
      $O_{out} = O_{out} \setminus \{(\alpha, \beta)\}$;
      $count := count + 1$;
      $m' := m' \cup \{X\_count\}$;
      $O := O \setminus \{(\alpha, \beta)\} \cup \{(\alpha, \beta \setminus \{x\} \cup \{X\_count\})\}$;
    end;
  $TEP := \wp(m') \setminus \{\varnothing\}$;
  for all $\gamma \in TEP$
    $O := O \cup \{(\gamma, \{x\})\}$;
    $split := split \setminus \{x\}$;
  end;
  $M := M \cup m'$;
end;

Figure 4. Algorithm Redundancy Generation (RG).

components can be taken into consideration as well as consuming the four main components of biodiesel: HDS gasoline, kerosene, K7 component (gasoline without sulfur), and FAME biocomponent. The biodiesel can be uploaded at any of the plants, transported to a target location by trucks, and finally downloaded. All of the plants have upper bounds on their available resources: 1500 tons of biodiesel is available in Dombóvár and 2000 tons in Pécs. A maximum of 935 tons biodiesel can be produced by blending at the plant in Pécs. Table 1 shows the inputs and the outputs as well as the reliability of activities.

■ REDUNDANCY IN PRODUCT SUPPLY SCENARIOS

The P-graph representation serves as a well established mathematical model to clarify the logical relations of the activities in synthesis problems and resultant supply scenarios unambiguously, which is essential to decide whether a scenario is redundant or not. For instance, if an activity has multiple preconditions, each of the preconditions needs to be satisfied for performing activity, which is a logical AND constraint. If an activity has more than one precondition, the activity node in the P-graph is represented by a horizontal bar with more than one incoming arcs. In Figure 1a the P-graph shows that "Precondition 1" AND "Precondition 2" are required for Activity.

If a target can be achieved by two or more activities, then any combination of them can be sufficient, which is a logical OR condition. When an entity can be an outcome of more than one activity, it is represented by a solid circle with multiple incoming arcs in the P-graph. The P-graph in Figure 1b shows that "Result" can be reached by "Activity 1" OR "Activity 2". This circumstance gives a chance to synthesize three alternative scenarios where "Result" is achieved, i.e., by performing "Activity 1" only, by "Activity 2" only, or by executing both Activity 1 and 2 in parallel.

In the theory of systems' reliability, two main structures of systems are considered: they are serial and parallel systems.[15] The serial systems symbolize structures where all of the activities have to be successful in order to achieve the overall target. Thus, the overall reliability of a sequential system is the product of

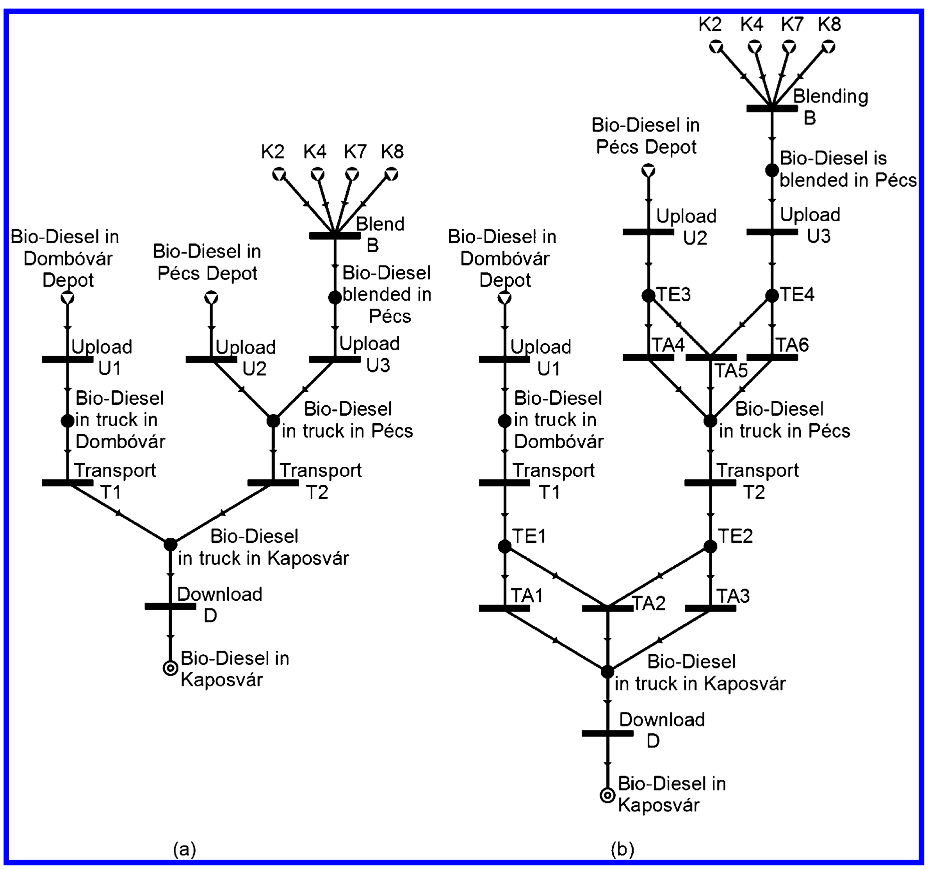

Figure 5. (a) Input of the algorithm RG, i.e., the initial or maximal structure of the illustrative example, and (b) output of the algorithm RG, i.e., redundancy extended maximal structure.

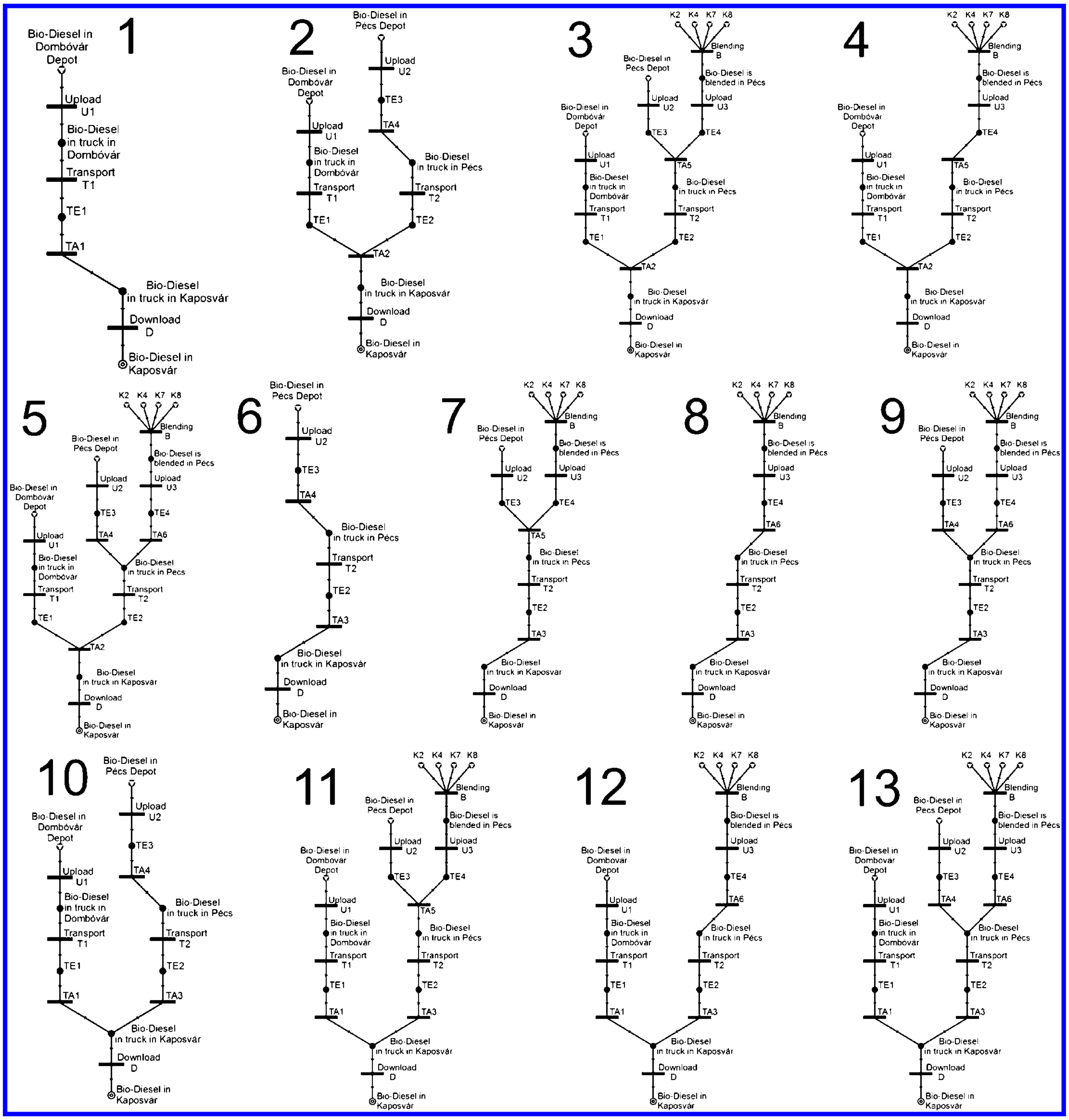

Figure 6. Alternative scenarios for the illustrative example.

individual reliabilities of activities in the sequence, i.e., the reliability is typically decreased by involving additional steps into the sequence. Parallel systems characterize structures where the overall target can be achieved if at least one of the parallel activities is successful. Let us consider the (100% − reliability) as risk. The overall risk of a parallel system is the product of the individual risks of activities which can be performed in parallel, i.e., the risk can be decreased by involving additional parallel activities into the system.

For complex process networks where activities are not redundant, the overall reliability can be calculated similarly to sequential systems, i.e., all of the activities have to be successful regardless of the network's topology. The previously presented OR relation can yield in numerous scenarios, either redundant or not, to be distinguished in reliability analysis. As an example, for the OR relation of two Activities, six plausible scenarios need to be differentiated; see Figure 2. In Figure 2a only Activity 1 produces the Result, Activity 2 does not take part in the production; in Figure 2b only Activity 2 provides the Result, Activity 1 does not contribute; in Figure 2c Activities 1 and 2 generate the required volume of the Result in cooperation; in Figure 2d both Activity 1 and 2 provide the required volume of the Result, i.e., redundantly in parallel; in Figure 2e Activity 1 produces the required q volume and Activity 2 produces a smaller v portion as well, i.e., partially redundantly; finally, Figure 2f represents the scenario when Activity 2 results in the required volume q and Activity 1 results in a smaller volume v, i.e., partially redundantly.

To be able to distinguish these six scenarios, the structural representation is extended by additional entity and activity type nodes; see Figure 3. The solid circle with symbol Σ represents the

Table 2. Overall Reliabilities of Alternative Supply Scenarios and the Activities Involved

structure	activities	overall reliability	redundancy (yes/no)
Str1	D, TA1, T1, U1	95.08%	no
Str2	D, TA2, T1, U1, T2, TA4, U2	98.84%	yes
Str3	D, TA2, T1, U1, T2, TA5, U2, U3, B	98.92%	yes
Str4	D, TA2, T1, U1, T2, TA5, U3, B	98.73%	yes
Str5	D, TA2, T1, U1, T2, TA4, U2, TA6, U3, B	98.66%	yes
Str6	D, TA3, T2, TA4, U2	95.08%	no
Str7	D, TA3, T2, TA5, U2, U3, B	96.92%	yes
Str8	D, TA3, T2, TA6, U3, B	92.23%	no
Str9	D, TA3, T2, TA4, U2, TA6, U3, B	90.38%	no
Str10	D, TA1, T1, U1, TA3, T2, TA4, U2	91.31%	no
Str11	D, TA1, T1, U1, TA3, TA5, U2, U3, B	93.09%	yes
Str12	D, TA1, T1, U1, TA3, T2, TA6, U3, B	88.58%	no
Str13	D, TA1, T1, U1, TA3, T2, TA4, U2, TA6, U3, B	86.80%	no

Table 3. Formulas for Calculating Overall Reliability of Alternative Supply Scenarios

structure	overall reliability	redundancy (yes/no)
Str1	$r_{U1} \times r_{T1} \times r_D = 95.08\%$	no
Str2	$(r_{U1} \times r_{T1} + r_{U2} \times r_{T2} - r_{U1} \times r_{T1} \times r_{U2} \times r_{T2}) \times r_D = 98.84\%$	yes
Str3	$\{r_{U1} \times r_{T1} + (r_{U2} + r_B \times r_{U3} - r_{U2} \times r_B \times r_{U3}) \times r_{T2} - r_{U1} \times r_{T1} \times [(r_{U2} + r_B \times r_{U3} - r_{U2} \times r_B \times r_{U3}) \times r_{T2}]\} \times r_D = 98.92\%$	yes
Str4	$(r_{U1} \times r_{T1} + r_B \times r_{U3} \times r_{T2} - r_{U1} \times r_{T1} \times r_B \times r_{U3} \times r_{T2}) \times r_D = 98.73\%$	yes
Str5	$(r_{U1} \times r_{T1} + r_B \times r_{U3} \times r_{U2} \times r_{T2} - r_{U1} \times r_{T1} \times r_B \times r_{U3} \times r_{U2} \times r_{T2}) \times r_D = 98.66\%$	yes
Str6	$r_{U2} \times r_{T2} \times r_D = 95.08\%$	no
Str7	$(r_{U2} + r_B \times r_{U3} - r_{U2} \times r_B \times r_{U3})) \times r_{T2} \times r_D = 96.92\%$	yes
Str8	$r_B \times r_{U3} \times r_{T2} \times r_D = 92.23\%$	no
Str9	$r_B \times r_{U3} \times r_{U2} \times r_{T2} \times r_D = 90.38\%$	no
Str10	$r_{U2} \times r_{T2} \times r_{U1} \times r_{T1} \times r_D = 91.31\%$	no
Str11	$[(r_{U2} + r_B \times r_{U3} - r_{U2} \times r_B \times r_{U3} \times r_{T2}) \times r_{U1} \times r_{T1})] \times r_D = 93.09\%$	yes
Str12	$r_B \times r_{U3} \times r_{T2} \times r_{U1} \times r_{T1} \times r_D = 88.58\%$	no
Str13	$r_B \times r_{U3} \times r_{U2} \times r_{T2} \times r_{U1} \times r_{T1} \times r_D = 86.80\%$	no

sum of the alternative activities results. The horizontal bar with notation "min" represents an activity which results the minimum volume of its inputs. In order to result the required volume by activity "min", each of the activities providing the inputs to "min" has to be sufficient in producing the required amount, i.e., redundancy is guaranteed. Finally, it is important to note that the proposed extension of the P-graphs, as illustrated in Figure 3, provides unambiguous and unique structural representation for each scenario visualized in Figure 2.

According to the P-graph framework, the input to the structure generation and optimization is the maximal structure,[1,2] which is a superstructure generated algorithmically for the Process Network Synthesis (PNS) problem of interest, e.g., by executing Algorithm MSG in software PNS Studio.[16] PNS Studio is a software package designed to solve problems in PNS. Process synthesis is the act of conceiving or determining the optimal structure of a process system of concern as well as the optimal types, configurations, and capacities of the functional units performing various operations within the system. For more details see refs 16 and 17.

Algorithm Redundancy Generation (RG) was developed for extending the maximal structure to clearly represent potentials for redundancy as described above. Algorithm RG in each of its iterations finds such an entity type node which has more than one input edges. The P-graph is extended at each of such nodes to denote potential cooperation and redundancy of those activities. As a result, a superstructure is generated involving each alternative scenario as its part. The algorithm Redundancy Generation is given in Figure 4. For better understanding formal definitions of a PNS problem, P-graph, maximal structure, and structural mappings are given in ref 18.

■ COMPUTATIONAL RESULTS

Taking into consideration the illustrative example, two nodes of entity type have more than one input edge, namely, both "Bio-Diesel in Truck in Kaposvár" and "Bio-Diesel in Truck in Pécs" can be provided by two activities. Algorithm RG extends the maximal structure by entities TE1, TE2, TE3, TE4, and activities TA1, TA2, TA3, TA4, TA5, TA6. Figure 5a shows the input of algorithm RG, i.e., the initial or maximal structure. Figure 5b depicts the extended maximal structure for the illustrative example generated by algorithm RG.

The redundancy extended maximal structure contains steps, which can guarantee the redundancy in the supply chain and increase its reliability consequently. Activities have reliability of 97.00−99.00%; see Table 1. The reliability of the supply scenarios ranges from 86.80% to 95.08% without redundancy and from 93.09% to 98.92% with redundancy.

Figure 6 depicts P-graph representations of alternative supply scenarios, i.e., solution structures for the case study resulted by algorithm SSG for the redundancy extended maximal structure, except those containing partial redundancy. There are 43 alternative solution structures, each of which is combinatorially feasible, 7 of them do not contain redundancy, and 6 of them involve fully redundant substructures, i.e., activities performed in parallel where duplicated production of a required volume of their targets is guaranteed.

Values of the parameters, algorithm RG, and the combinatorially feasible structures generated for the case study were detailed. After the alternative feasible structures were generated by algorithm SSG, overall reliabilities of the scenarios have been calculated. Tables 2 and 3 list overall reliabilities of alternative supply scenarios and the formulas for reliability calculations, respectively.

Note that based on the redundancy-extended superstructure, each scenario is generated by the original P-graph algorithms without modifying them, e.g., by executing Algorithm SSG in software PNS Studio.[16] Computational results illustrate that applying the proposed algorithm novel process structures involving redundant substructures are generated, yielding increased reliability values.

■ CONCLUDING REMARKS

The current work presents an addition to a former methodology[1−5] to model supply scenarios formally and to algorithmically synthesize optimal and n-best suboptimal supply scenario by the P-graph framework. The framework traditionally provides algorithms for modeling the structure of supply networks and generating alternative structures as well as optimizing them based on cost or profit. The algorithms presented herein broaden the applicability of the framework for reliability analysis and redundancy generation. An algorithm is introduced to extend the initial or maximal structure prior to the optimization to incorporate such elements in the network, which can guarantee redundancy and thus increase the overall reliability of a supply scenario. Based on the resultant redundancy extended maximal structure each scenario can be generated by the original algorithms of the P-graph framework.

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Notes

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