

Optimization software for solving vehicle assignment problems to minimize cost and environmental impact of transportation

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A method and software are proposed for optimal assignment of vehicles to transportation tasks in terms of total cost and emission. Each task is given by a set of attributes to be taken into account in the assignment; and this is also the case for each vehicle. The overall mileage is calculated as the sum of the lengths of all the routes to be travelled during, before, after, and between the tasks (Deaulniers et al., 1998; Baita et al., 2000). Cost and emission are assigned to the mileages of each vehicle type.

1. Introduction

According to EPA reports, transportation accounted for approximately 29 % of the total US greenhouse gas (GHG) emissions in 2006. Moreover, transportation is the fastest-growing source of US GHGs, corresponding to 47 % of the net increase in total US emissions since 1990. Transportation is also the largest end-use source of CO₂. Nevertheless, these estimates do not include emissions from additional processes involved in the lifecycle of the transportation systems, such as the extraction and refining of fuel and the manufacture of vehicles, which are also significant sources of domestic and international GHG emissions (US EPA, 2009).

The two major means for reducing the pollution caused by transportation are changing the personal behavior of the end users and improving the quality of transporting networks, for which the economic crisis of these years serves as a motivation.

Optimization software for supporting human decisions is essential to implement the two major means mentioned above. Presented herein is an algorithmic method for calculating the optimal assignment of vehicles to transportation tasks supported by software tools at each step.

2. Problem definition

The vehicle assignment problem of interest is defined by a set T of tasks and a set S of resources. For each $P_i \in T$ task, i.e., trip to be performed, the starting time $t_s(P_i)$ and location $l_s(P_i)$ as well as the ending time $t_e(P_i)$ and location $l_e(P_i)$ are given.

Table 1: Tasks to be completed for the example

Task $P_i \in T$	Starting time $t_s(P_i)$	Starting location $l_s(P_i)$	Ending time $t_e(P_i)$	Ending location $l_e(P_i)$	Potential Resources $A(P_i)$
Trip P1	7:00	Tihany	7:40	Almadi	R1, R2
Trip P2	8:00	Veszprem	9:20	Fehervar	R1, R3
Trip P3	8:40	Veszprem	9:40	Tihany	R1, R2

Table 2: Available resources for the example

Resources $R_k \in S$	Location $l(R_k)$	Cost $c(R_k)$ [€/km]	CO ₂ emission $e(R_k)$ [g/km]	Maximum speed $v_{\max}(R_k)$ [km/h]
Truck R1	Tihany	0.6	375	90
Truck R2	Fehervar	0.5	400	90
Truck R3	Kenese	0.4	300	90

For each $R_k \in S$ resource, i.e., vehicle, the actual location $l(R_k)$, cost $c(R_k)$, CO₂ emission $e(R_k)$, and maximum speed $v_{\max}(R_k)$ are specified. Moreover, for each $P_i \in T$ task, the elements of set $A(P_i)$ of those resources potentially capable to performing task P_i are listed. Each task is considered to require the full capacity of a single resource. Tables 1 and 2 show the details of the example with three tours and three vehicles. The distances between pairs of the locations in the example are denoted in Figure 1.

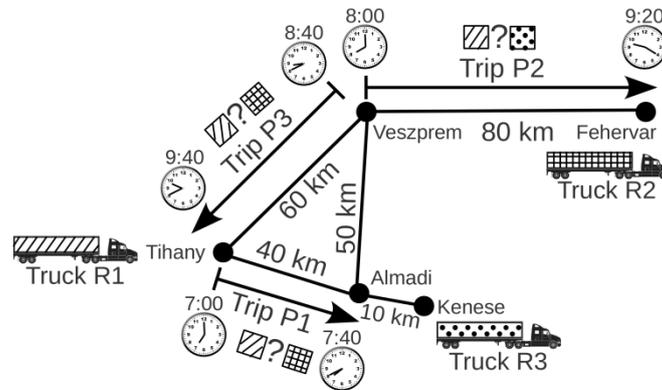


Figure 1: Map of the tasks, resources, and locations for the example

3. Methodology

The vehicle assignment problem is transformed into the corresponding process-network synthesis (PNS) problem. The P-graph representation of the PNS problem provides an easily discernable structural model and the basis for effective solution by combinatorial accelerations as well (Friedler et al., 1992, 1993, 1995, 1996 and Friedler 2009, 2010). The proposed approach guaranties that the resultant solution is globally optimal; in addition, it yields the n -best feasible assignments.

A PNS problem is given by a triplet (P, R, O) , where set P contains the final targets to be achieved; set R , the initially available resources; and set O , the candidate activities to form a network and reach each of the final targets by deploying any of the available resources. Each activity is defined by its preconditions and outcomes. A precondition can be the availability of a resource or an outcome of another activity. In any transportation network, the location and time of availability of resources and outcomes are essential, and thus, they are well defined. For each vehicle assignment problem, the tasks to be completed serve as the final targets of the network in set P , the resources are listed in set R , and set O of candidate activities involves the tasks performed by each appropriate resource or movement of resources from one location to another.

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input:  $(T, S, A, t_s, t_e, l_s, l_e, d, l, v_{max})$  assignment problem
output:  $(P, \mathcal{R}, \mathcal{O})$  process synthesis problem
begin
   $\mathcal{P} := T; \mathcal{R} := S; \mathcal{O} := \emptyset;$ 
  for all  $P_i \in T$  do
    for all  $R_k \in A(P_i)$  do
       $\mathcal{O} := \mathcal{O} \cup \{(\{R_k\}, \{R_k-l_s(P_i)-t_s(P_i)\}), (\{R_k-l_s(P_i)-t_s(P_i)\}, \{R_k-l_e(P_i)-t_e(P_i), P_i\})\};$ 
    end for
    for all  $P_j \in T$  do
      for all  $R_k \in A(P_i) \cap A(P_j)$  do
        if  $t_e(P_i) \leq t_s(P_j)$  and  $\frac{d(l_e(P_i)-l_s(P_j))}{t_s(P_j)-t_e(P_i)} \leq v_{max}(R_k)$  then
           $\mathcal{O} := \mathcal{O} \cup \{(\{R_k-l_e(P_i)-t_e(P_i)\}, \{R_k-l_s(P_j)-t_s(P_j)\})\};$ 
        end if
      end for
    end for
  end for
end

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Figure 2: Algorithm for transforming a vehicle assignment problem into the corresponding process-network synthesis problem

Figure 2 shows the algorithm for constructing the corresponding process synthesis problem for a vehicle assignment problem. It generates three classes of activities. In the first class, an activity is assigned to the movement of each resource from its initial location to each task, which the resource can perform. In the second class, an activity is assigned to each task-resource pair, where the task can potentially be completed by the resource. And finally, in the third class, activities are defined for the movement of the resources between two tasks if and only if the following three conditions hold. First, both tasks can be performed by a single activity. Second, the first task precedes the second task. Third, the resource can travel to from the ending location of the first task to the starting location of the second task in time by its maximal speed. Table 3 lists the candidate activities generated by the algorithm for the example.

Table 3: Activities to be considered in process synthesis for the example

Activity	Precondition	Result	Duration [min]	Cost [€]	CO ₂ emis. [kg]
O1	R2	R2_Veszprem_8:40	80	40	32.0
O2	R2	R2_Tihany_7:00	140	70	56.0
O3	R1	R1_Tihany_7:00	0	0	0.0
O4	R1	R1_Veszprem_8:40	60	36	22.5
O5	R1	R1_Veszprem_8:00	60	36	22.5
O6	R3	R3_Veszprem_8:00	60	24	18.0
O7	R2_Almadi_7:40	R2_Veszprem_8:40	60	25	24.0
O8	R1_Almadi_7:40	R1_Veszprem_8:40	60	30	22.5
P1ByR1	R1_Tihany_7:00	R1_Almadi_7:40, P1	40	24	15.0
P1ByR2	R2_Tihany_7:00	R2_Almadi_7:40, P1	40	20	16.0
P2ByR1	R1_Veszprem_8:00	R1_Fehervar_9:20, P2	80	48	30.0
P2ByR3	R3_Veszprem_8:00	R3_Fehervar_9:20, P2	80	32	24.0
P3ByR1	R1_Veszprem_8:40	R1_Tihany_9:40, P3	60	36	22.5
P3ByR2	R2_Veszprem_8:40	R2_Tihany_9:40, P3	60	30	24.0

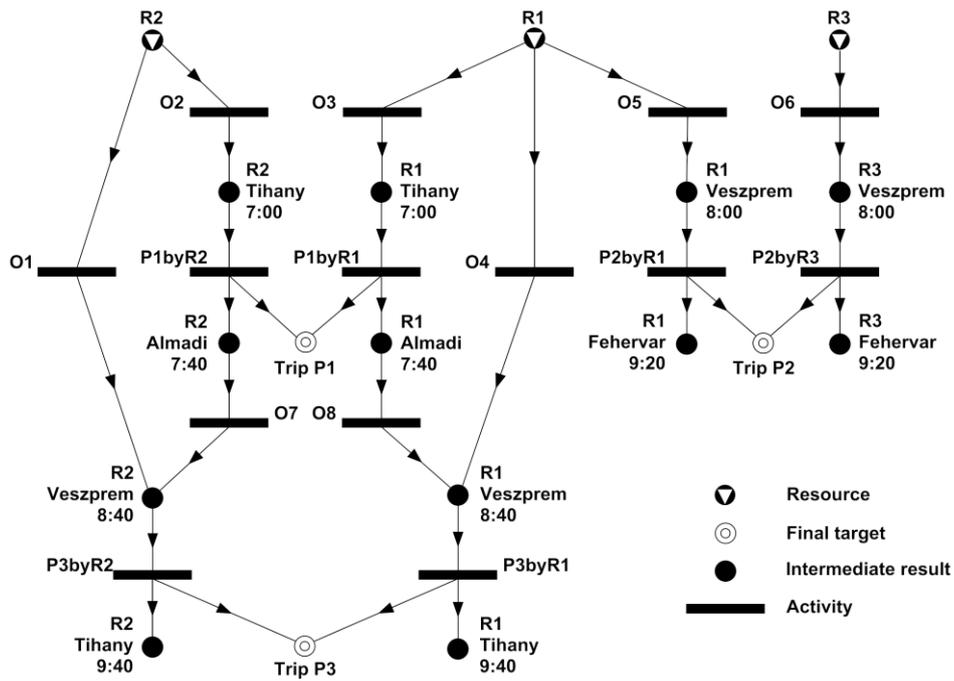


Figure 3: P-graph representation of the maximal structure for the example

Table 4: Five best vehicle assignments for the example

Solution	Assignments			Mileage			Total mileage [km]	Cost [€]	CO ₂ emission [kg]	Max running time [min]
	P1	P2	P3	R1	R2	R3				
#1	R1	R3	R1	150	0	140	290	146	10.20	160
#2	R1	R3	R2	40	140	140	320	150	11.30	140
#3	R2	R1	R2	140	290	0	430	201	16.20	240
#4	R2	R3	R2	0	290	140	430	218	15.90	180
#5	R2	R3	R1	120	180	140	440	229	17.20	240

In the P-graph framework, algorithm MSG produces the maximal structure for the PNS problem (Friedler et al., 1993). This maximal structure serves as the input to the generation and solution of the mathematical model by algorithm ABB (Friedler et al., 1996). Figure 3 presents the maximal structure for the example. Algorithm ABB yields the optimal and predetermined n -best suboptimal networks. Table 4 contains the five best vehicle assignments for the example.

4. Results and discussion

The alternative feasible solutions generated by algorithm ABB can be ranked according to multiple criteria. Such criteria can be, e.g., the costs, emissions, and running times of the vehicles. Typically, the cost and emission serve as the objectives; and the maximum running time of a single vehicle, as the constraint because of regulations. Even if both cost and emission are usually related to the mileage of a vehicle, the optimal or n -best assignments may differ for these two objectives due to the reduced emissions from modern vehicles of high costs; see Solutions #4 and #5 for the example in Table 4.

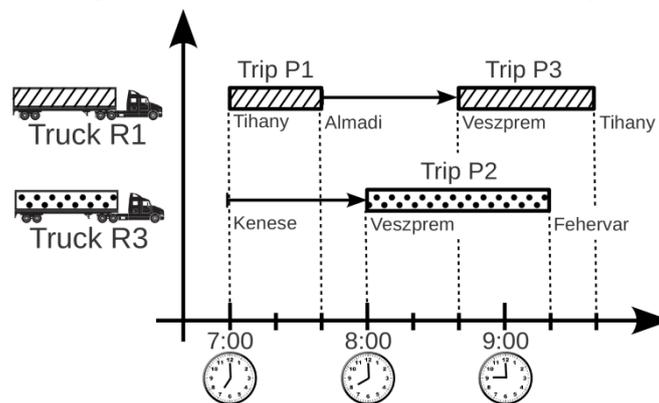


Figure 4: Optimal assignment of vehicles to trips for the example

Note that the optimal solution is identical from the economical and ecological points of view depicted in detail in the Gantt diagram in Figure 4. In contrast, Table 4 highlights

that the rankings of assignments by their costs, emissions, and maximum running times result in different orders; and thus, it is important to generate alternative assignments.

5. Conclusions

An algorithmic method has been proposed for generating optimal and n -best suboptimal solutions for vehicle assignment problems. The method has been crafted by reformulating an assignment problem as a PNS problem and solving the resultant problem by algorithms and software of the P-graph framework. The potential of the proposed method has been illustrated by solving an example in which the optimal and suboptimal alternative assignments of the different ranked order emerge by regarding the costs, emissions, and running times of the vehicles as the objective function.

Acknowledgement

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